

Examiners' Report Principal Examiner Feedback

Summer 2024

Pearson Edexcel International Advanced Level In Pure Mathematics P4 (WMA14) Paper 01

Introduction

This WMA14_01 paper was a very good test of the specification. Questions 1, 2, 3, 5c and 8a proved to be a very good source of marks for all candidates with the rest providing differentiation at all grades. The paper was of appropriate length with little evidence of students rushing to complete the paper.

Points to note for future exams

- Candidates should show their method when presenting solutions as just writing down answers, especially when incorrect, can lead to the loss of all the marks.
- Candidates should take care when presenting solutions to questions and should clearly show all steps when solving multi- step problems. For example, in question 9 on integration,

candidates wrote down
$$\int \frac{1}{u^2}$$
 or $\int \frac{1}{u^2} dx$ when they should have written $\int \frac{1}{u^2} du$

Question 1

This question on integration by parts was very accessible to a well-prepared candidate, many of whom scored all 5 marks.

Reasons for a loss of marks included:

- incorrect integration of $\cos 3x$ with incorrect signs and multiplication by 3 common
- a failure to apply the limits of integration correctly to the whole integrated expression with the limit of 0 being frequently omitted

Question 2

It has been mentioned in previous examiners reports that the use of a diagram would aid candidates understanding of a question and this was such a case. For many candidates this was a straightforward question and there were lots of fully correct responses. Unfortunately subtracting vectors when they should have been added, and vice versa, had huge consequences.

In part (a) candidates simply had to use $\overrightarrow{OA} + \overrightarrow{AB}$ to find \overrightarrow{OB} . The answer could be given in coordinate or vector form and many achieved one of the correct versions. There were the usual slips when adding but both marks were lost when candidates attempted a subtraction.

In part (b) candidates were required to use the fact that OC was perpendicular to BC.

Successful candidates used the fact that $\overrightarrow{OC}.\overrightarrow{BC} = 0$ to set up an equation in a which could be easily solved. Errors and a general misunderstanding of the geometry in this part of the question were common, many of which could have been addressed by a well labelled diagram. Reasons for a loss of marks here were:

- a failure to find vector \overrightarrow{BC} via $\overrightarrow{OC} \overrightarrow{OB}$
- a failure to understand which vectors were perpendicular or else setting $\overrightarrow{OC}.\overrightarrow{BC}$ equal to something other than 0 such as 1

Question 3

This was a very standard and familiar question on implicit differentiation. The well-prepared candidate knew how to differentiate the terms $3y^2$ and 2xy with respect to x. The gradient of the curve at (2, 5) could then be found and from there, the equation of the normal to the curve at (2, 5). Slips and errors were common, the most noteworthy being:

- a failure to differentiate the 9 thus reaching $24x^2 6y\frac{dy}{dx} + 2y + 2x\frac{dy}{dx} = 9$
- a failure to fully differentiate the 2xy term reaching $2x\frac{dy}{dx}$
- finding the equation of the tangent rather than the normal
- leaving the equation in the form y = mx + c, rather than the required form 13x + 53y 291 = 0

Question 4

This was a question on related rates of change set within the context of a segment of a circle. Candidates struggled with the area of the segment, even though it is a WMA11 topic.

In part (a) the form of the answer for $\frac{dA}{d\theta}$ was given although this did not make the question any easier for most. Many only found the area of the sector or the triangle and guessed the value for K. There was only a minority of fully correct solutions.

Part (b) was more familiar and most candidates picked up the mark for stating $\frac{d\theta}{dt} = 0.1$ and

 $\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$. Candidates who did not do part (a) could still pick up 3 of the 4 marks for

using the chain rule to find a value or expression for $\frac{dA}{dt}$

Question 5

This question was based around the area under a parametric curve. Aspects that caused issues here were:

- the area R was above the curve so the calculation (total area) -(area under curve) had to be applied
- a misunderstanding between x coordinates and values of the parameter t

Part (a) was very straightforward and most candidates achieved the correct t values of 1 and 2 for the values of a and b. A few then mistakenly went on to use those correct t values, by stating that a was 3 and b was 8 which were the x values at the points, not the t values.

In part (b) candidates were required to find the area of region R. A healthy clue was given by asking them to show that it could be written in the form $M - k \int_a^b \frac{t+1}{t(3-t)} dt$. This was a rather discriminating question with many assuming that the total area was 1 and not $1 \times (8-3) = 5$. Most prepared candidates could prove the $4 \int_1^2 \frac{t+1}{t(3-t)} dt$ aspect via

$$\int_{1}^{2} y \frac{\mathrm{d}x}{\mathrm{d}t} \, \mathrm{d}t = \int_{1}^{2} \frac{2}{t(3-t)} \times (2t+2) \, \mathrm{d}t.$$

Elements of part (c) were very straightforward. Most candidates scored the 3 marks for the partial fractions in (c)(i). The first mark in (c) (ii) was also obtained by most who were able to integrate the terms in $\frac{1}{t}$ and $\frac{1}{(3-t)}$. The last two marks were discriminating and involved accurately drawing together all aspects of the question and simplifying the log terms.

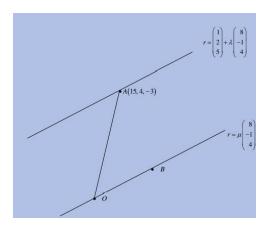
Question 6

This was the second question on vectors and another in which it would have been beneficial to draw a diagram. Parts (a) and (b) were relatively straightforward with part (c) being very discriminating even at grade A standard.

In part (a) candidates were required to use the fact that $|\overrightarrow{OA}| = 5\sqrt{10}$ where A was a point on the given line to show that $81\lambda^2 + 52\lambda - 220 = 0$. Most knew what to do, although a huge error was committed by some who used just the gradient vector as opposed to the point $(1+8\lambda, 2-\lambda, 5+4\lambda)$ when finding $|\overrightarrow{OA}|$. Errors were rare but seen when candidates failed to correctly square $5\sqrt{10}$ or else not showing sufficient steps in the proof. Part (b) involved solving the given equation and using the values of λ to find the two positions of A. The point (-15, 4, -3) was given and it was important to show all steps leading to this answer. A score of 1, 0, 1 was common when candidates assumed they could just write down a given answer.

Part (c) could be attempted by various methods including via the scalar product or the cosine rule. Good candidates drew a diagram and could easily determine what was required. Once the angle was found via scalar product or the cosine rule, the area of the triangle could be found via the formula $\frac{1}{2}OA \times OB \sin AOB$. A very common error in part (c) worthy of note was in determining the angle AOB. The most straightforward way was via the scalar product of $\begin{pmatrix} -15 \\ 4 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ -1 \\ 4 \end{pmatrix}$, but many if not most candidates, tried to find vector OB using its length

which, although a valid method, did introduce aspects that could lead to slips in calculations.



Question 7

This question, set in context, required candidates to solve and use a first order separable differential equation.

Part (a) should have been familiar to a well-prepared candidate as a fairly standard model was used. Once integrated, the fact that t=0, x=0 could be used to find the constant of integration. The introduction of a constant k in the initial equation caused some difficulty but prepared candidates managed at least 4 marks out of the seven. Major causes of loss of marks were:

- a failure to add a constant of integration, thus losing $-\frac{1}{3} \ln k$
- loss of the $-\frac{1}{3}$ when integrating to $-\frac{1}{3}\ln(k-3x)$

Further maths candidates attempted this part via an integrating factor approach which was acceptable.

In part (b), the long-term current could be used to find the value of k. This could be attempted by two different ways. Using the equation was only possible if the equation was of a suitable form, in other words it having a limit as $t \to \infty$. It could also be determined via the initial differential equation $\frac{dx}{dt} = k - 3x$ as $\frac{dx}{dt} \to 0$ when x = 7, and so $k - 3 \times 7 \to 0$. Part (b) was discriminating but very good candidates could show their skills here.

In part (c) candidates were required to find the value of t at which x = 5. Marks were only awarded if the equation used was of the correct form and solvable. Again, it proved to be a discriminating question, and the very best candidates were able to shine.

Question 8

The modal score here was 5 out of 8 marks. Part (a) was a reasonably straightforward application of the binomial expansion whereas part (b) required problem solving at a fairly sophisticated level.

Most prepared candidates achieved all 4 marks in part (a). Marks were lost when:

- candidates took out a common factor of 8 rather than $8^{\frac{4}{3}}$
- candidates made bracketing errors and found $\frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6} \times \frac{3}{8} x^3$ rather than $\frac{\frac{4}{3} \times \frac{1}{3} \times \left(-\frac{2}{3}\right)}{6} \times \left(\frac{3}{8} x\right)^3$
- candidates didn't fully show all the steps leading to the form $A 8x + \frac{x^2}{2} + Bx^3 + ...$

Part (b) was discriminating at the very highest grades. Most candidates could score only the first mark for setting up the contradiction. Fully correct solutions here were rare, but when seen, were eloquent and showed an excellent understanding of both proof and analysis. Solutions based entirely upon calculator technology were not allowed so candidates who argued that the cubic $8-16x+8x^2+\frac{x^3}{24}=0$ at x=-194 would only score the first mark.

Ouestion 9

The last question on WMA14 should be discriminating at the highest grades and this was no exception. The two marks for correctly differentiating $x = \tan^2 u$ were accessible to most, as was the mark for obtaining the limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$. Other marks were more demanding with

weaker students being unsure of whether they were working with $\arctan\left(\sqrt{x}\right)$ or $\frac{1}{\tan\left(\sqrt{x}\right)}$. As

the form of the answer was given, many achieved this but from fudged working. The very best

of candidates easily transformed
$$\frac{x^{-\frac{1}{2}}}{\left(1+x\right)\left(\arctan\left(\sqrt{x}\right)\right)^{2}} dx \text{ into } \frac{\left(\tan u\right)^{-1}}{\left(1+\tan^{2} u\right)u^{2}} 2\tan u \sec^{2} u du$$

which via the identity $1 + \tan^2 u = \sec^2 u$ could be written in the form $\frac{\left(\tan u\right)^{-1}}{\sec^2 u \times u^2} 2 \tan u \sec^2 u \, du$

The first two marks in part (b) was very accessible for those that obtained the correct limits. Again, as in many previous questions, the final mark was only awarded for those who had legitimately obtained the correct answer in part (a).